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2 SEM TDC STS M 1 (N/O)

2016

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 48
Pass Marks : 14

Time : 2 hours

1. Choose the correct answer : 1×6=6

(a) If $A = \{1, 2, \{3, 4\}, 5\}$, then which of the following statements is incorrect?

(i) $\{3, 4\} \in A$

(ii) $\{\{3, 4\}\} \subset A$

(iii) $\{3, 4\} \subset A$

(iv) None of the above

(b) Which of the following is not equivalent to $A \subset B$?

(i) $A - B = \phi$

(ii) $A \cap B = A$

(iii) $A \cup B = B$

(iv) None of these

(c) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(d) According to Cauchy's root test, $\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = l > 1$ means that the series Σu_n is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) convergent to 1 only

(e) The first derivative of the function x^8 w.r.t. another function x^3 is

(i) $\frac{3}{8}x^5$

(ii) $\frac{8}{3}x^5$

(iii) $24x^5$

(iv) None of the above

(f) The value of $\int_0^{\pi/2} \sin^6 x \, dx$ is

(i) $5\pi/64$

(ii) $5\pi/32$

(iii) $5/32$

(iv) None of the above

2\. (a) If S and T are subsets of real numbers, then show that $(S \cup T)' = S' \cup T'$. 3

(b) Show that a set is closed iff its complement is open. 3

3\. Answer any *two* of the following : 6×2=12

(a) Define a bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left(\frac{1}{a_n}\right) = \frac{1}{\overline{\lim}\, a_n}, \text{ if } \overline{\lim}\, a_n > 0$$

1+5=6

(b) State Cauchy's first theorem on limits. Using the theorem, show that

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

1+5=6

(c) What is monotonic sequence? Show that the sequence $\{a_n\}$ defined by

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right), \ n \geq 1 \text{ and } a_1 > 0$$

converges to 3.

1+5=6

4. (a) Show that the function $f(x) = x^2 - 6x$ is increasing for $x > 3$. 2

(b) Show that $D^n(x^n) = n!$ 2

5. Answer any *two* of the following :

(a) If $\sin y = x \sin(a + y)$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

5

(b) If $z = \frac{x^2 y^2}{x + y}$, then prove that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$$

5

(c) State and prove Leibnitz theorem. 5

6. Answer any *two* of the following :

(a) If $f(x) = f(a + x)$, then prove that

$$\int_0^{na} f(x)\, dx = n \int_0^a f(x)\, dx$$

5

(b) Evaluate : 5

$$\int_0^2 \int_0^{\sqrt{4+x^2}} \frac{dx\, dy}{4 + x^2 + y^2}$$

(c) If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

and deduce the value of I_5. 5

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. State which of the following statements are true and which are false : 1×7=7

 (a) A function of the type $f(x, y) = 0$ is called implicit function.

 (b) If $x = \phi(a)$, $y = \psi(t)$, then
 $$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

 (c) The value of
 $$\int_0^{\pi/2} \sin^6 x\, dx$$
 is $\frac{5\pi}{32}$.

 (d) The union of two closed sets is not a closed set.

 (e) The set of all integers is countable.

 (f) Every bounded sequence has a limit point.

 (g) According to d'Alembert's ratio test, $\lim\limits_{n \to \infty} \frac{u_{n+1}}{u_n} = l < 1$ means that the series Σu_n is convergent.

2. (a) If $x^3 + y^3 - 3axy = 0$, then show that
 $$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$
 5

 (b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that
 $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$
 6

 Or

 (c) Show that

 (i) $D^n(x^n) = n!$

 (ii) $D^n\left(\frac{1}{x+a}\right) = \frac{(-1)^n n!}{(x+a)^{n+1}}$ 3+3=6

 (d) Define maxima and minima of a function. Find for what values of x, the expression $f(x) = 2x^3 - 15x^2 + 36x + 10$ is maximum and minimum respectively, and hence find the maximum and minimum values. 2+5=7

Or

(e) State Leibnitz theorem for the nth derivative of the product of two functions. Using the theorem or otherwise, show that $x^2y_2 + xy_1 + y = 0$ for $y = a\cos(\log x) + b\sin(\log x)$. 2+5=7

3. (a) Show that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Using this property or otherwise, prove that

$$\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x}\,dx = 0$$

2+4=6

(b) Prove that

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \frac{(2m)!}{\{2^m m!\}^2}\frac{\pi}{2}$$

5

Or

(c) Find : 5

$$\int_1^2 \int_0^x \frac{dx\,dy}{x^2 + y^2}$$

(d) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$

5

(e) Find Laplace transform of the function $(t+2)^2 e^t$. 3

4. (a) Define Cartesian product of two sets. Find $A \times B$ if $A = \{x \mid x = 1, 2\}$, $B = \{y \mid y = x + 2\}$. 1+3=4

(b) Show that a countable union of countable sets is countable. 4

Or

(c) Prove that a set is closed iff its complement is open. 4

(d) What are infimum and supremum of a set? Find the infimum and supremum of the sets $S_1 = \{2, 4, 6, 8\}$ and $S_2 = \left\{\frac{1}{n}, n \in N\right\}$. 5

Or

(e) If S, T are subsets of real numbers, then—

(i) show that $(S \cap T)' \subseteq S' \cap T'$;

(ii) give an example to show that $(S \cap T)'$ and $(S' \cap T')$ may not be equal. 2½+2½=5

(f) Define a field stating clearly its properties. 5

Or

(g) Define a set function. For the finitely additive set function f defined on the field F, prove that

$$f(A \cup B) + f(A \cap B) = f(A) + f(B), \forall \ A, B \in F$$

1+4=5

5. (a) Define convergent, divergent and oscillatory series. Give an example of a series used in statistical analysis, which is convergent. 3+1=4

(b) What is a monotonic sequence? If $x_n = \dfrac{3n-1}{n+2}$, then prove that the sequence $\{x_n\}$ is monotone increasing and bounded. 5

Or

(c) Show that the sequence $\{S_n\}$, defined by the recursion formula $S_{n+1} = \sqrt{3S_n}$, $S_1 = 1$ converges to 3. 5

(d) Define Cauchy's root test and hence test for the convergence of the series where the general term is $\left(1+\dfrac{1}{\sqrt{n}}\right)^{-n^{3/2}}$. 2+2=4

Or

(e) Show that the series

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

is convergent. 4

(f) Show that the series

$$\Sigma \frac{3.6.9.\cdots.3n}{7.10.13.\cdots.(3n+4)} x^n, x > 0$$

converges for $x \le 1$ and diverges for $x > 1$. 5

Or

(g) Prove that every absolutely convergent series is convergent. Show that for any fixed values of x, the series $\sum_{n=1}^{\infty} \dfrac{\sin nx}{x^2}$ is convergent. 3+2=5

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