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2 SEM TDC STS M 1

2014

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×8=8

(a) $\log_e x$ and e^x are

(i) both increasing functions

(ii) decreasing and increasing function respectively

(iii) both decreasing functions

(iv) increasing and decreasing function respectively

(b) If $f(c)$ is the maximum of f and $f'(c)$ exists, then

(i) $f'(c) = 0$

(ii) $f'(c) > 0$

(iii) $f'(c) < 0$

(iv) $f'(c) = k$

where k is a non-zero constant

(c) Laplace transform (LT) of $F(t) = e^{-2t}$ is

(i) $1/(s+2)$

(ii) $1/(s-2)$

(iii) $1/s^2$

(iv) $2/s$

(d) Value of $\int_a^b f(t)\,dt$ is always same as the value of

(i) $-\int_b^a f(x)\,dx$

(ii) $\int_{-a}^{-b} f(t)\,dt$

(iii) $-\int_b^a f(-t)\,dt$

(iv) $\int_a^b f(-t)\,dt$

f is an odd function

(e) A limit point of a set may

(i) not be a member of the set

(ii) always be a member of the set

(iii) always be a finite number

(iv) always exist

(f) If $A = \{a, b, c\}$ and $B = \{c, d\}$, then $A \cup B$ is

(i) $\{a, b, c, d\}$

(ii) $\{a, b, c, c, d\}$

(iii) $\{c\}$

(iv) a null set

(g) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(h) If the series $\sum_{n=1}^{\infty} u_n$ is convergent, then

$$k \sum_{n=1}^{\infty} u_n$$

(i) is also convergent

(ii) is divergent

(iii) may or may not converge

(iv) converges for some values of k only

where k is a non-zero constant

2. (a) Find $\dfrac{dy}{dx}$ if $x = a\cos^3\theta$, $y = a\sin^3\theta$. Show that it is a parametric form of the equation $x^{2/3} + y^{2/3} = a^{2/3}$ and hence find dy/dx for the second equation. 3+3=6

(b) Find the maximum profit (in lakh Rs) that a company can make, if the profit function is given by

$$p(x) = 41 - 72x - 18x^2$$

4

(c) Differentiate x^5 w.r.t. x^2. 2

(d) (i) Show that the function

$$f(x) = 4x^3 - 6x^2 - 72x + 30$$

is strictly increasing in the intervals $(-\infty, -2)$ and $(3, \infty)$ and, strictly decreasing in $(-2, 3)$. 6

Or

(ii) If $u = \log(x^2 + y^2)$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

3. Answer any *three* :

(a) Prove that

$$\int_{-a}^{a} f(x) = 0 \text{ if } f(-x) = -f(x)$$
$$= 2\int_0^a f(x)\,dx, \text{ if } f(-x) = f(x)$$

4

(b) Write down the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, n is positive integer and hence evaluate $\int_0^{\pi/2} \sin^6 x\,dx$. 4

(c) If $u = x + y$ and $v = \dfrac{x}{x+y}$, find $J\left(\dfrac{x, y}{u, v}\right)$. 4

(d) Define Laplace transform of a function $F(t)$ and mention two of its important properties. 4

4. (a) Evaluate

$$\underset{n \to \infty}{\text{Lt}} \left[\frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \cdots + \frac{n}{(n+n)^2}\right]$$

with the help of definite integral. 6

Or

(b) Evaluate

$$\int_0^a \int_0^{\sqrt{a^2 - y^2}} \sqrt{(a^2 - x^2 - y^2)}\, dy\, dx$$

6

5. (a) Define a set and a derived set. Illustrate with example(s). 4

(b) If Cartesian product of two sets A and B is given by

$$A \times B = \{(p, r), (q, r), (p, s), (q, s)\}$$

find A, B and $A \times (A \cap B)$. 4

(c) In a survey of 75 students in a school, 39 like to play cricket and 42 like to play football. Also each student likes to play at least one of the two games. How many students like to play both cricket and football? 5

(d) (i) Prove that a countable union of countable sets is countable. 5

Or

(ii) Prove that a set is closed iff its complement is open.

6. (a) Define a bounded sequence and show that the sequence $\{S_n\}$ where

$$S_n = 1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \cdots + \frac{1}{\lfloor n}, \ \forall n \in N$$

is bounded. 5

(Continued)

(b) Using D'Alembert's ratio test, show that the series

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{\lfloor x}, \lambda > 0$$

is convergent. Interpret the meaning of the result when x is a random variable. 4+1=5

(c) (i) Prove that a sequence can't converge to more than one limit point. State Leibnitz test and test the convergence of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$ 4+4=8

Or

(ii) Show that a positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent iff $p > 1$. Define Cauchy's root test and mention about the decision taken when the test fails. 5+3=8

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