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2 SEM TDC STS M 1

2013

(May)

STATISTICS

(Major)

Course : 201

(Mathematics for Statistics—I)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×8=8

(a) If $|x|$ is the absolute value of a real number x, then

(i) $|x| \geq 0$

(ii) $|x| \leq 0$

(iii) $|x| < 0$

(iv) None of the above

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(b) The set $N = \{a, b, c, d\}$ of natural numbers is equivalent to

(i) $\{2, 4, 6, 8\}$

(ii) $\{1, 3, 5, 7\}$

(iii) $\left\{\frac{1}{2}, \frac{1}{3}, \frac{5}{4}, \frac{7}{5}\right\}$

(iv) $\{-1, -3, -5, -7\}$

(c) The function $f(x) = x^2$, $x \in (0, \infty)$ is

(i) strictly increasing

(ii) strictly decreasing

(iii) non-increasing

(iv) non-decreasing

function of x.

(d) The third derivative of the function $y = e^{ax}$ is

(i) $a^3 e^{ax}$

(ii) $6e^{ax}$

(iii) $3a^3 e^{ax}$

(iv) e^{ax} / a^3

(e) The value of $\int_0^{\frac{\pi}{2}} \sin^4 x\, dx$ is

(i) $\frac{3\pi}{16}$

(ii) $\frac{3\pi}{8}$

(iii) $\frac{3}{16}$

(iv) None of the above

(f) $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ for all values of c such that

(i) $a < c < b$

(ii) $c > 0$

(iii) $c > a$

(iv) $c < d$

(g) A convergent sequence is

(i) always bounded

(ii) bounded above only

(iii) bounded below only

(iv) neither bounded above nor bounded below

(h) According to d'Alembert's ratio test, $\underset{n \to \infty}{\text{Lt}} \frac{u_{n+1}}{u_n} = l < 1$ means that the series Σu_n is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) convergent, to 1 only

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2. *(a)* Find the interval(s) in which the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ is increasing. 4

(b) If $L\{F(t)\} = f(s)$ is the Laplace Transform (LT) of the function $F(t)$, then find

$$L\left\{t^n + \frac{1}{6}\right\} \text{ and } L\{e^{at}\}$$ 4

(c) What are infimum and supremum of a set? Find infimum and supremum of the sets

$$S_1 = \{2, 4, 6, 8\} \text{ and } S_2 = \left\{\frac{1}{n}, \; n \in N\right\}$$ 4

(d) Define convergent, divergent and oscillatory series. Give an example of a series used in statistical analysis, which is convergent. 3+1=4

3. *(a)* *(i)* State Leibnitz theorem for the *n*th derivative of the product of two functions. Using the theorem or otherwise, show that

$$x^2 y_2 + x y_1 + y = 0$$

for $y = a\cos(\log x) + b\sin(\log x)$. 2+5=7

(ii) A random sample of size 400 is to be collected from two districts *A* and *B*. The cost of collecting *m* units from *A* and *n* units from *B* is given by the cost function

$$f(m, n) = 3m^2 + mn + 2n^2 + 10$$

Use the method of Lagrange's multiplier to determine *m* and *n* in such a way that the cost is minimum. 7

Or

(b) *(i)* If $\sin y = x\sin(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

Also find partial derivatives u_{xx}, u_{xy} and u_{yy} for

$$u = x^3 + 3x^2 y + 3xy^2 + y^3$$ 4+3=7

(ii) Define maxima and minima of a function $f(x)$. Find the points at which

$$f(x) = x(x - 1)^2$$

assumes maximum and minimum value respectively. 2+5=7

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4. (a) (i) Show that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Using this property or otherwise, prove that

$$\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\sin x \cos x}\,dx = 0$$ 2+5=7

(ii) Define Jacobian of transformation and give an example of its application in statistics. If

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$ 2+5=7

Or

(b) (i) Show that

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\,dx = \frac{1\cdot3\cdot5\cdot\ldots(m-1)\cdot1\cdot3\cdot5\cdot\ldots(n-1)}{2\cdot4\cdot6\cdot\ldots(m+n)}\cdot\frac{\pi}{2}$$

when both m and n are even integers. 9

(ii) Evaluate : 5

$$\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} (x^2+y^2)\,dx\,dy$$

5. (a) When a set is said to be bounded? Prove that every infinite bounded set has a limit point. 1+5=6

Or

(b) Define partition of a set. Mention the properties of partition of a set. 1+5=6

6. (a) Define countable and uncountable sets. Show that the set R of real numbers in [0, 1] is uncountable but the set Q of rational numbers in [0, 1] is countable. 2+6=8

Or

(b) Define union and intersection of sets. Give example. If S and T are subsets of real numbers, then show that

$$(S\cup T)' = S'\cup T'$$ 3+5=8

7. (a) (i) Show that the sequence $\{a_n\}$ defined by

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right),\ n\geq 1,\ a_1 > 0$$

is convergent and it converges to 3. 5+2=7

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(ii) Prove that a necessary condition for convergence of an infinite series Σu_n is that $\text{Lt}\, u_n = 0$ as $n \to \infty$. Using the condition, comment on the convergence of the series

$$\sum \frac{1}{n} \text{ and } \sum \frac{n}{n+1}$$ 4+3=7

Or

(b) *(i)* Prove that every bounded sequence has a limit point. If

$$a_n = (-1)^n n, \quad n \in N$$

then show that

$$\lim a_n = -\infty$$ 5+2=7

(ii) Give a comparison test for positive term series Σu_n and Σv_n. Test the convergence of the series

$$\sum \{(n^3 + 1)^{\frac{1}{3}} - n\}$$ 3+4=7

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