www.prepnext.com

Notes Prepared By:
RAKESH AGARWAL
M.Com, MBA, FIII
E-mail: messagerakesh@gmail.com
WhatsApp No: 8486118428

Coaching Available for: *Bank P.O./ Clerk, SSC, Railway, MAT, CA, CS, B.Com and M.Com. For details, call 8486118428 or email at info@prepnext.com*

Security Analysis & Portfolio Management

UNIT – 3

Q: What do you understand by Single Index Model?

(www.prepnext.com)

Ans:

The Markowitz method was conceptually very sound and theoretically very elegant, but it had two major shortcomings. The first is the problem of computational complexity. The second problem is that it assumes that all the risk & return characteristics can be explained by the covariance of the securities' returns with the returns of other securities. Thus, changes in factors, such as the growth rate of the economy or the inflation rate, are not accounted for directly.

These considerations have led to various simplifications and extensions of the model. One such simplification is the **Single Index**

www.prepnext.com

Model, developed by Sharpe. Single index model simplifies the mathematical calculations required by Markowitz Model, and also reduces the number of required inputs.

The major assumption of Sharpe's single index model is that all the covariances of security returns can be explained by a single factor. This factor is called the Index, hence the name "Single Index Model". The Single Index Model relates returns to a single factor, thus eliminating Markowitz's complex method of calculating all covariances of all assets in the portfolio. This means that for n securities we require 'n' statistical inputs. So, instead of 499500 correlation coefficients required for 1000 securities in Markowitz Model, we need just 1000 values. One version of the model, called the **Market Model**, uses a market index such as the Sensex as the factor, although in principle, any factor that influences security returns can serve as the index.

Sharpe postulated that the return on security should be related to a single Market Index. It has the following implications:-

i) Theoretically, the Market Index would consist of all the securities traded on the market. However, a popular average can be treated as a surrogate for the Market Index.

ii) Single Market Index will reduce and simplify the work involved in compiling elaborate matrices of variances as between individual securities; because any movements in securities could be attributed to movements in the single underlying factor being measured by the market price.

The Single Index Model (Market Model) states that the return on an individual stock over a given time period is related to the return on the same period that is earned on a market index such as the Sensex or the Nifty. Thus, if the market goes up, then the stock goes up and if the market goes down, then the stock goes down.

www.prepnext.com

Thus, $r_i = \alpha_i + \beta_i r_m + \varepsilon_i$

Where,

r_i = return on stock

α_i = intercept term

β_i = slope term,

r_m = return on the market index , and

ε_i = random error term.

[This defines a line with intercept α_i and slope β_i , with ε_i being the deviations from the line for the individual returns. This line is called the Security Characteristic Line (SCL])

Under the single index model, the return of a security consists of two components – (i) the return due to the market; and (ii) the return independent of the market. The β variable in the above equation indicates the sensitiveness of the stock return to the change in the market return. The variable α in the above equation is the non-market component i.e., return independent of the market. It denotes the return of the security even when the market return is zero.

Assuming that the slope term is positive, the above equation indicates that the higher the return on the market index, the higher is the return on the individual stock likely to be. The expected value of random error term is zero.

Example:

Consider a stock which has r_m = return on the market index = 12 %, α_I = 3% and β_i = 1.5, then the expected return on the stock is 3 % + 1.5 * 12 % = 21 %.

Similarly if the market index's return is – 8%, then the return on the security is expected to be 3 % + 1.5 *– 8% = – 9%

www.orepnext.com

Q: Explain Single Factor Model in brief. (*www.prepnext.com)*

Ans.:

According to single factor model, the asset price depends on a single factor, say Gross National Product or industrial production or interest rates, money supply, and so on.

In general, a single factor model can be represented in equation form as follows:

$$r_i = \alpha_i + b_i F + \varepsilon_i$$

Where,

r_i = return on stock i

α_i = intercept term

b_i = Security's sensitivity to change in the factor

F = Actual return on the factor

ε_i = random error term (Unexplained variable)

Q: Explain Two Factor Model in brief. *(www.prepnext.com)*

Ans.:

In case of two factors, denoted by F_1 and F_2, each security will have two sensitivities, b_{i2} and b_{i2}. The security returns are generated by the following model:-

$$r_i = \alpha_i + b_{i1} F_1 + b_{i2} F_2 + \varepsilon_i$$

Where,

r_i = return on stock

α_i = Intercept term (alpha coefficient of security)

ε_i = random error term

www.prepnext.com

Q: Explain Multiple Factor Model in brief. *(www.prepnext.com)*

Ans:

Factor models or index models assume that the return on a security is sensitive to the movement of various factors or indices. The market model (which is a single factor model) assumes that there is one factor – the return on a market index. However, in estimating expected returns, variances and covariances for securities, the multiple factor models are more useful than the market model. This is because actual security returns are sensitive to more than movements in a market index. Multiple factor model attempts to capture the major economic forces that systematically move the prices of all securities.

Empirical work suggests that a number of variables should be taken into account for asset pricing. A multiple factor model can be represented in equation form as follows:

$$r_i = \alpha_i + b_{i1} F_1 + b_{i2} F_2 + b_{i3} F_3 + \ldots + \varepsilon_i$$

Each of the middle terms in the equation is the product of returns on a particular economic factor and the given stock's sensitivity to that factor.

The factors are the underlying economic forces that influence the stock market. Several factors appear to have been identified as being important. In particular, the researchers have identified the following factors:

- Change in the level of industrial production
- Change in the interest rate
- Change in the inflation rate
- The level of personal consumption
- The level of money supply in the economy

Please provide your suggestions/ feedback at: *8486118428*

www.prepnext.com

Q: Explain Capital Asset Pricing Model in detail.

(www.prepnext.com)

Ans.:

The model was developed by Jack Treynor, William Sharpe, John Lintner and Jan Mossin independently, building on the earlier work of Harry Markowitz on diversification and modern portfolio theory. The CAPM is an economic model that describes how securities are priced in the marketplace. It predicts the relationship between the expected return and risk of individual securities and portfolios in the capital markets.

ASSUMPTIONS:

1. Investors make choices on the basis of risk and return.
2. Investors have homogeneous expectations of risk and return.
3. Investors have identical time horizon.
4. Information is freely and simultaneously available to investors.
5. There is risk-free asset, and investors can borrow and lend unlimited amounts at the risk-free rate.
6. There are no taxes, transaction costs, restrictions on short sales, or other market imperfections.
7. Total asset quantity is fixed, and all assets are marketable and divisible.
8. Individuals are risk averse.
9. The investor's objective is to maximise the utility of wealth.
10. Purchase and sales by a single investor cannot affect prices. This means that there is perfect competition where investors in total determine prices by their actions.
11. The investor can sell short any amount of any shares.
12. Investors hold diversified portfolios. So, they will only require a return for the systematic risk of their portfolios, since unsystematic risk has been removed and can be ignored.

www.prepnext.com

13. A Single-period transaction horizon (standardised holding period) is assumed by the CAPM in order to make comparable the returns on different securities. A return over six months, for example, cannot be compared to a return over 12 months.

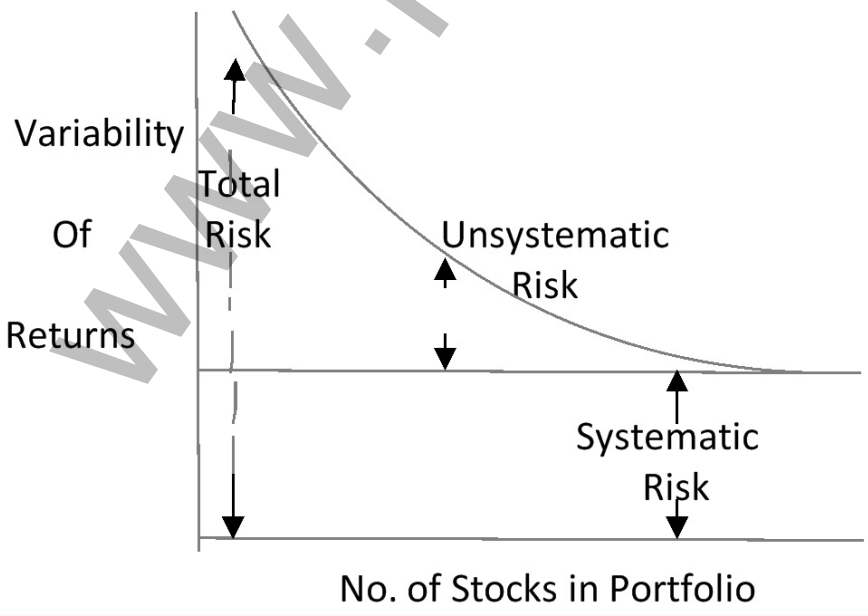
CAPITAL ASSET PRICING MODEL:

The key element of the model is that it separates the risk affecting an asset's return into two categories. The first type is called unsystematic, or company-specific, risk. The second kind of risk, called systematic risk, is due to general economic uncertainty.

Investors attempt to reduce risk through diversification of investment. Unsystematic risk can be mitigated through diversification. The real risk of a security is the market risk (systematic risk) which cannot be eliminated through diversification. It is risk that affects all investments or classes of investments. This is measured by the beta coefficient of the security.

The total risk of portfolio is reduced with increase in the number of stocks, as a result of decrease in the unsystematic risk distributed over number of stocks in the portfolio, as shown in the figure below:

Risk Exposure of Portfolio



www.prepnext.com

The Capital Asset Pricing Model gives the nature of the relationship between the expected return and the systematic risk of a security. The linear relationship between the return required on an investment and its systematic risk is represented by the CAPM formula:

Expected Return

= Risk Free Rate + Risk Premium

= Risk -Free Rate + [Beta x (Market Return – Risk-Free Rate)]

OR

$E(R_i) = R_f + ß_i [E(R_m) – R_f]$

Where,

$E(R_i)$ = Expected Rate of Return on a risky security (i)

R_f = Risk free rate of retutn

$ß_i$ = Beta (Market Sensitivity Index) of security i

R_m = Expected market rate of return

Thus, if the risk-free rate is 3 %, the beta (risk measure) of the stock is 2 and the expected market return over the period is 10%, the expected return of stock is 17% (3% + 2 (10% - 3%)).

The CAPM formula states that the return on each risky security or portfolio is simply the risk free rate plus some risk premium for investing in the risky security. The risk premium is the return provided by the "market" less the risk free return. In other words, it is the amount of extra return on top of the risk free rate that an investor should be compensated for exposing himself to securities riskier than the risk free asset.

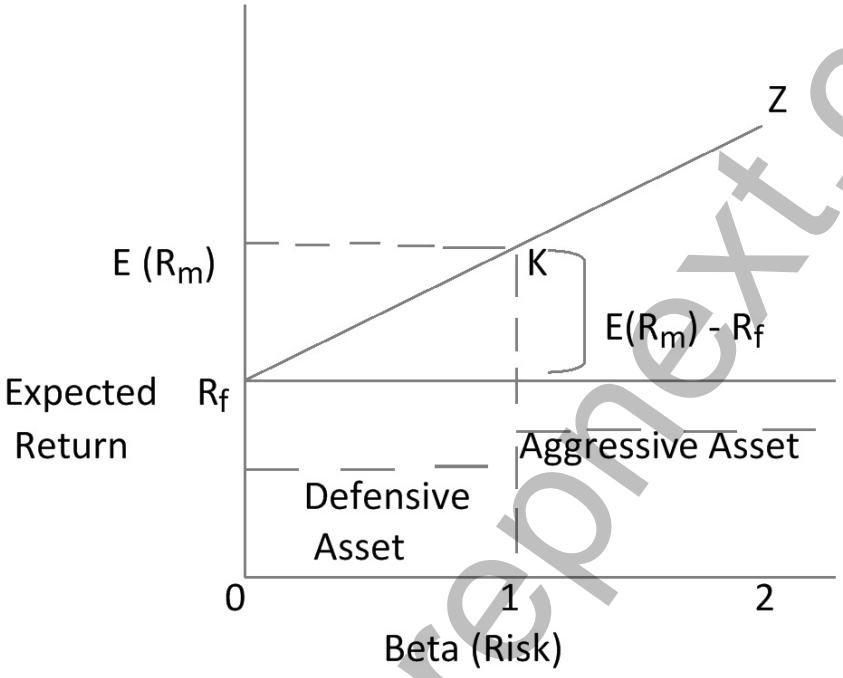
Example: *Calculating the Required Return Using the CAPM*
If the risk-free rate of a Treasury bill is 4%, and the return of the stock market has averaged about 12%, what is the required return of a stock that has a beta of 1.4?

By using the CAPM formula, shown above, we find that:
Required Return= 4 % + [1.4 x (12% – 4 %)]
= 4% + 1.4 x 8% = 4% + 11.2% = 15.2%

www.prepnext.com

So if this stock returns only 13% in the past few years, then it has a greater risk than is justified by its return compared to the general market.

Capital Asset Pricing Model is represented by a CAPM line drawn on risk-return space. The following figure portrays the CAPM model graphically.



Point K represents the market portfolio and point R_f the risk less rate of return. The CAPM intersects the vertical axis at the risk less rate R_f.

The above figure shows that the return expected from portfolio or investment is a combination of risk free return plus risk premium. If this expected return does not meet or beat the required return, then the investment should not be undertaken. For example, if Security A has an expected return of 5%, but based on the CAPM the expected return *should* be 6%, then you should not buy Security A because for the same level of risk you can find other investments with an expected return of 6%.

www.prepnext.com

The general idea behind CAPM is that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free (R_f) rate in the formula and compensates the investors for placing money in any investment over a period of time. The other half of the formula represents risk and calculates the amount of compensation the investor needs for taking on additional risk. This is calculated by taking a risk measure (beta).

ADVANTAGES OF THE CAPM

The CAPM has several advantages over other methods of calculating required return:

1) It considers only systematic risk, reflecting a reality in which most investors have diversified portfolios from which unsystematic risk has been essentially eliminated.

2) The assumption of a single-period transaction horizon appears reasonable from a real-world perspective, because even though many investors hold securities for much longer than one year, returns on securities are usually quoted on an annual basis.

3) The CAPM focuses on the market risk, and makes the investors think about the riskiness of the assets in general.

4) The CAPM has been useful in the selection of securities and portfolios. Securities with higher returns are considered to be undervalued and attractive for buy. The below normal expected return yielding securities are considered to be overvalued and suitable for sale.

5) In the CAPM, it has been assumed that investors consider only the market risk. Given the estimate of the risk free rate, the beta of the stock and the required market rate of return, one can find out the expected returns for a firm's security.

www.prepnext.com

SHORTCOMINGS OF CAPM

The CAPM suffers from a number of disadvantages and limitations:

1. The model assumes that all investors have access to the same information and agree about the risk and expected return of all assets. This is not the case in real life,
2. The model does not appear to adequately explain the variation in securities' returns. Empirical studies show that low beta stocks may offer higher returns than the model would predict.
3. The model assumes that given a certain expected return investors will prefer lower risk to higher risk and conversely given a certain level of risk will prefer higher returns to lower ones. It does not allow for investors who will accept lower returns for higher risk. *(casino players)*
4. The model assumes that there are no taxes or transaction costs, although this assumption is not valid in the real life.
5. The model assumes that all assets are infinitely divisible as to the amount which may be held or transacted.
6. The model assumes just two dates, so that there is no opportunity to consume and rebalance portfolios repeatedly over time.
7. CAPM focuses attention only on systematic (market related) risk. However, total risk has been found to be more relevant.
8. Investors do not seem to follow the postulation of CAPM and do not diversify in a planned manner.
9. Beta (systematic risk) coefficient is unstable, varying from period to period depending up on the method of compilation. They may not be reflective of true risk involved. Historical evidence of the tests of Beta showed that they are unstable and they are not good estimates of future risk.

www.prepnext.com

10. The assumption of a single-period time horizon is at odds with the multi-period nature of investment appraisal.
11. In reality, it is not always possible for investors to borrow at the risk-free rate.
12. In order to use the CAPM, values need to be assigned to the risk-free rate of return, the return on the market, or the equity risk premium (ERP), and the equity beta.
13. The market portfolio should in theory include all types of assets that are held by anyone as an investment. In practice, such a market portfolio is unobservable and people usually substitute a stock index as a proxy for the true market portfolio.
14. The model does not appear to adequately explain the variation in stock returns. Empirical studies show that low beta stocks may offer higher returns than the model would predict.
15. CAPM assumes that all investors will consider all of their assets and optimize one portfolio. This is in sharp contradiction with portfolios that are held by investors: humans tend to have fragmented portfolios (or rather multiple portfolios – for each goal one portfolio)
16. The CAPM is based on expectations about the future. Expectations cannot be observed but we do have access to actual returns. Hence, empirical tests and data for practical use tend to be based almost exclusively on historical returns.
17. The historical data regarding the market return, risk free rate of return and betas vary differently for different periods. The various methods used to estimate these inputs also affect the beta value. Since the inputs cannot be estimated precisely, the expected return found out through the CAPM model is also subjected to criticisms.

www.prepnext.com

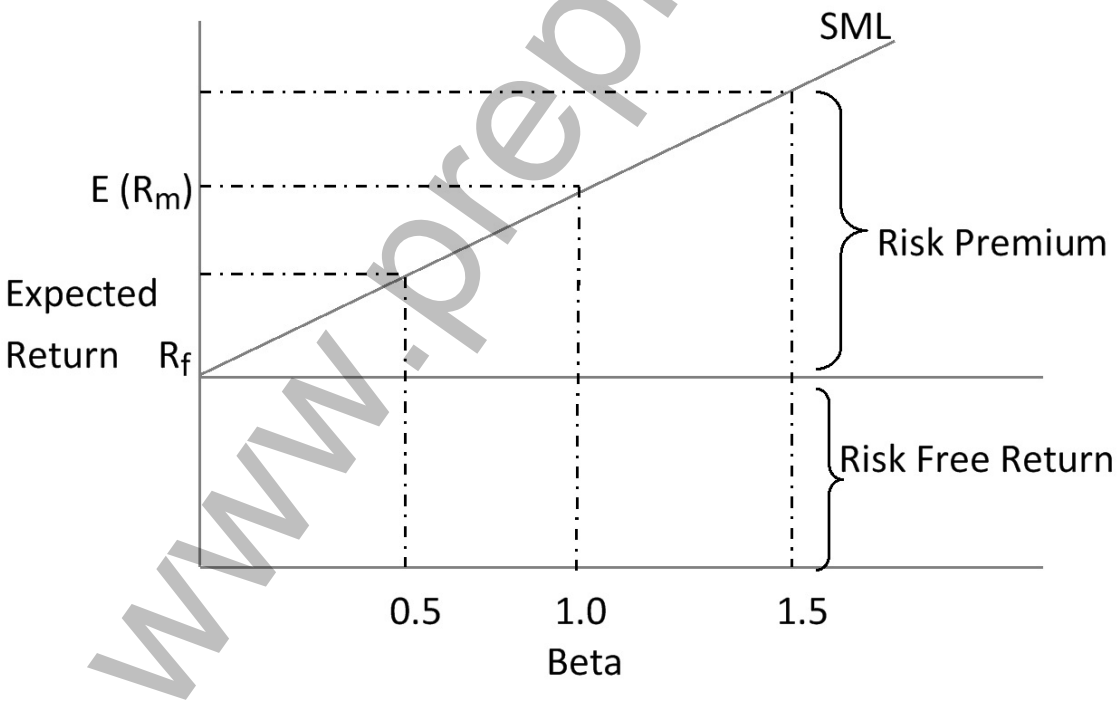
Q: What do you know about Security Market Line (SML)?

(www.prepnext.com)

Ans.:

Security Market Line describes the expected return of all assets and portfolios of assets in the economy. The risk of any stock can be divided into systematic risk and unsystematic risk. Beta (ß)is the index of systematic risks. In case of portfolio involving complete diversification, where the unsystematic risk tends to zero, there is only systematic risk measured by Beta. The expected return on any asset or portfolio, whether it is efficient or not can be determined by SML by focusing on Beta of securities. The higher the Beta for any security the higher must be its equilibrium return.

The SML expresses the basic theme of the CAPM i.e. expected rate of return incraeses linearly with risk, as measured by beta. It is the graphical representation of the CAPM model. It can be drawn as follows:



The above figure shows that the return expected from portfolio or investment is a combination of risk free return plus risk premium.

www.prepnext.com

The CAPM has shown the risk and return relationship of a portfolio in the following formula:

$$E(R_i) = R_f + ß_i (R_m - R_f)$$

Where, $E(R_i)$ = Expected rate of return on any individual security or portfolio of securities.

R_f = Risk free rate of return

R_m = Expected rate of return on market portfolio

$ß_i$ = Market sensitivity index of individual security or portfolio of securities.

The SML is an upward sloping straight line with an intercept at the risk free return securities and passes through the market portfolio. The upward slope of the line indicates that greater expected returns accompany higher levels of Beta. In equilibrium each security or portfolio lies on the SML.

SML validates the claim that systematic risk is the only important ingredient in determining expected returns and that non-systematic risk plays no role. In other words, the investor gets rewarded for bearing systematic risk. It is not total variance of returns that affects expected returns but only that part of variance in return that cannot be diversified away.

Q: What do you understand by Capital Market line?

(www.prepnext.com)

Ans.:

The Capital Market Line defines the relationship between total risk and expected return for portfolios consisting of the risk free asset and the market portfolio. CML generates a line on which efficient portfolios can lie.

The CML says that the expected return on a portfolio is equal to the risk free rate plus a risk premium. The relationship between the

www.prepnext.com

return and risk of any efficient portfolio on the capital market line can be expressed in the form of the following equation:

$$\bar{r}_p = r_f + (\bar{r}_m - r_f)\, \partial_p / \partial_m$$

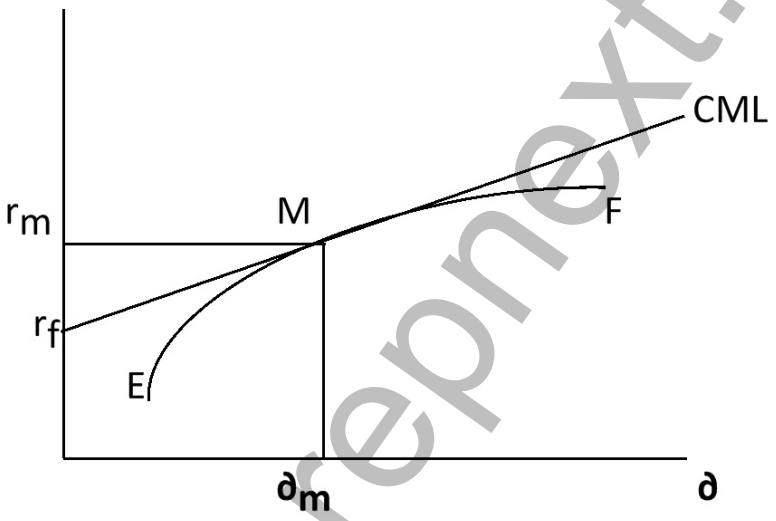
Where, r_f = risk free rate;

r_m = return on market portfolio;

∂_p = standard deviation of the return on the portfolio;

∂_m = standard deviation of the return on market portfolio

Graphically, the CML can be drawn as below:



The Capital Market Line

EF is the efficient frontier, M is the market portfolio and the line tangent to the efficient frontier and joining the risk free rate r_f with the market portfolio M and going beyond is the Capital Market Line (CML).

The risk free rate compensates investors for the time value of money (it is the price of time) while the risk premium compensates investors for bearing risk. The risk premium is equal to the market price of risk times the quantity of risk for the portfolio (as measured by the standard deviation of the portfolio).

$$(\bar{r}_m - r_f)\, \partial_p / \partial_m$$

www.prepnext.com

The term $(r_m - r_f)$ is the expected return of the market beyond the risk free return. It is a measure of the reward for holding the risky market portfolio rather than the risk-free asset. The term ∂_m is the risk of the market portfolio.

Thus the slope of the CML measures the reward per unit of risk. It determines the additional return needed to compensate for a unit change in risk. It is also called the market price of risk.

Thus, the expected return on an efficient portfolio is:
(Expected Return) = (price of time) + (Price of risk) (Amount of risk)

Q; Differentiate between Capital Market Line and Security Market Line. (www.prepnext.com)

Ans.:

1) In CML expected return of portfolio is taken on y-axis whereas in case of SML expected return of securities is taken on y – axis.

2) In CML standard deviation of portfolio is taken on x-axis whereas in SML Beta of security is taken on x-axis.

3) Only market portfolio and risk free asset lies on the CML whereas every security lies on the SML.

4) CML shows the relation between portfolio expected return and portfolio standard deviation and helps investors in their capital allocation problem, while the SML shows the relation between expected return and beta and helps investors in security selection and individual asset pricing.

5) Slope of CML = $[E(Rm) - R_f]$ / Standard deviation of m
Slope of SML = $[E(R_m) - R_f]$ / Beta of m = $E(R_m) - R_f$
Where $E(R_m)$ = Return on the market

www.prepnext.com

Q: Explain Arbitrage Pricing Model. (www.prepnext.com)

Ans.:

Stephen Ross presented the Arbitrage Pricing Theory (APT) in 1976. APT is a multi-factor model for determining the required rate of return which means that it takes into account a number of economy wide factors that can affect the security prices. Instead of correlating each security with only the market portfolio (one factor), this model correlates each security with an appropriate series of macro-economic factors (e.g., inflation, industrial production, interest rates, etc). APT starts with the assumption that security returns are related to an unknown number of unknown factors. These factors can be GDP (Gross Domestic Product), market interest rate, the rate of inflation or any other random variable that impacts security prices. Then, assuming that decision-makers take advantage of all arbitrage opportunities to hold portfolios that offer higher returns Ross built a model that explained an asset's risk and expected (or required) return in terms of its sensitivity to each of these basic economic factors.

The Arbitrage Theory is based on the following assumptions:

- **i)** The investors have homogeneous beliefs/ expectations.
- **ii)** The investors are risk averse and utility maximisers
- **iii)** The markets are perfect so that factors like transaction costs are not relevant.
- **iv)** The security returns are generated according to a factor model.
- **v)** Risk-returns analysis is not the basis.

According to APT, an investor will explore the possibility of forming an arbitrage portfolio to increase the expected return on his current portfolio without increasing its risk. An arbitrage opportunity arises if an investor can construct a zero investment portfolio with no risk, but with a positive profit. Since no investment is required, an investor can create large positions, in long and short, to secure large levels of profits. An arbitrage portfolio does not require any additional commitment of funds.

www.prepnext.com

Ross has proposed an approach to explain the pricing of risky assets. Ross's model, the Arbitrage Pricing Theory (APT) asserts that asset prices are determined through an arbitrage relationship. The APT is based on the premises that two or more securities or portfolios that provide the same pay-offs to their investors are same and must therefore sell at the same price. That is the law of one price. Put differently, if there are two securities that have the same risk but different expected returns, investors will arbitrage, or eliminate, these differences by buying the security with the higher expected return (lower price), and selling the one with the lower expected return (higher price). This process of buying and selling the two securities by investors will cause the price of the security of the higher expected return to rise relating to the one with the lower expected return. This trading activity will continue until the two securities have the same expected returns.

Limitation:

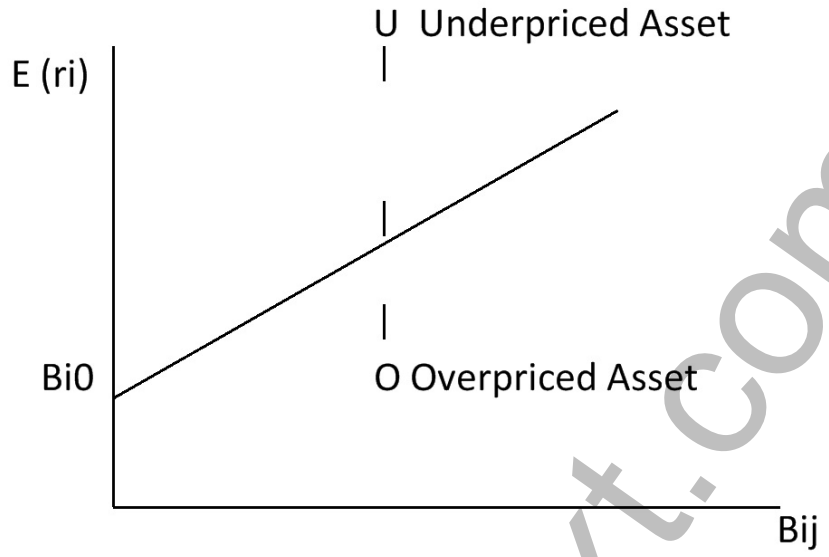
APT is a factor model, that starts with the assumption that security returns are related to an unknown number of unknown factors. It does not specify what these factors are. Unlike CAPM, APT does not rely on measuring the performance of the market. Instead, APT directly relates the price of the security to the fundamental factors driving it. The problem with this is that the theory in itself provides no indication of what these factors are, so they need to be empirically determined.

ARBITRAGE MECHANISM

Suppose there are two assets O and U with expected return-risk characteristics as shown in the figure below. Both the assets have the same level of risk but different expected returns. That is, assets O and U violate the 'law of one price' since two assets with the same level of risk are trading at different prices and offer different expected returns. The APT considers all assets, such as O and U, that are in the same risk class to be perfect substitutes and should be traded at the same price in equilibrium.

www.prepnext.com

Figure : The APT Model and Arbitrage



Where, Bi0 represents either risk-free return (if available) or the return on zero beta security.

Bij = Sensitivity or beta coefficient that measures the responsiveness of the security/ asset to changes in the factor under consideration.

E (ri) = Expected return on security/ asset i

This offers an arbitrage opportunity – a chance for riskless profit. One can see that an asset O is relatively overpriced and an asset U relatively underpriced. He can make a riskless profit by short selling the overpriced asset and utilising the proceeds in buying the underpriced asset. That is, go short on asset O and long on asset U. This strategy does not involve any initial investment since we assume that proceeds of the short sales can be utilised immediately and can be used to buy other assets. At the same time the investor will be ensured of a profit equivalent to the difference between the expected returns of the two assets. This process of arbitrage increases the demand for asset U and supply of asset O, which results in increase in the price of asset U and decrease in the price of asset O. This process of arbitrage goes on until both the assets are traded at the same price and offer the same expected return. That is, both assets lie on the arbitrage price line, ensuring equilibrium risk-return relation for all the securities.

www.prepnext.com