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5 SEM TDC PHY M 1

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(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×6=6

(a) The residue of

$$\frac{z^4}{(z-1)^4(z-2)(z-3)}$$

at $z = 1$ is

(i) 0

(ii) $\frac{525}{8}$

(iii) $\frac{175}{16}$

(iv) $\frac{175}{64}$

(b) In the general solution of second-order differential equation

$$(D^2 - 2\alpha D + \alpha^2)\, y = 0$$

one term contains $e^{\alpha x}$, then its second term will be constant times

(i) $e^{-\alpha x}$

(ii) $xe^{-\alpha x}$

(iii) $xe^{\alpha x}$

(iv) $\frac{1}{x} e^{\alpha x}$

(c) What is the argument of the quotient of two complex numbers?

(i) Quotient of their arguments

(ii) Product of their arguments

(iii) Sum of their arguments

(iv) Difference of their arguments

(d) If n is a positive integer, then the value of the integral

$$\int_{-1}^{+1} P_n(x)\, (1 - 2xh + h^2)^{-1/2}\, dx$$

is

(i) $\frac{2}{2n+1}$

(ii) 0

(iii) $\frac{2h^{-n}}{2n+1}$

(iv) $\frac{2h^n}{2n+1}$

(e) The value of

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

is

(i) $\frac{\pi^2}{2}$

(ii) $\frac{\pi^2}{3}$

(iii) $\frac{\pi^2}{6}$

(iv) $\frac{\pi^2}{8}$

(f) $\beta(m+1, n) + \beta(m, n+1)$ is equivalent to

(i) $\beta(m+1, n+1)$

(ii) $\beta(m-1, n-1)$

(iii) $\beta(m, n)$

(iv) $\frac{\beta(m, n)}{m+n}$

2. (a) Find the distance which an object falls under gravity in t seconds, if it starts from rest. Establish the differential equation first. 2

(b) Show that

$$P_n(-x) = (-1)^n P_n(x)$$

2

(c) If $f(z) = u + iv$ is an analytic function and $\vec{F} = v\hat{i} + u\hat{j}$ is a vector, then show that div. $\vec{F} = 0$ and curl $\vec{F} = 0$ are equivalent to Cauchy-Riemann equations. 2

(d) Find the residue of

$$\frac{z}{(z-a)(z-b)}$$

at infinity. 2

(e) If

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le \dfrac{\pi}{2} \\ \pi - x & \text{for } \dfrac{\pi}{2} \le x \le \pi \end{cases}$$

express it by a sine series. 2

(f) Expand e^{ax} in a series of sines of multiples of x in the interval $(0 < x < \pi)$. 2

3. (a) Use the Frobenius' method to obtain the solutions of $y'' + w^2 y = 0$. 5

Or

Solve the initial value problem : 5

$$y'' + y' - 2y = 0$$

$$y(0) = 4$$

$$y'(0) = -5$$

(b) Prove that

$$\int_{-1}^{+1} P_n(x)\, P_n(x)\, dx = \frac{2}{2n+1}\, \delta_{mn}$$ 5

(c) Solve : 5

$$y'' + y' - 2y = e^x$$

(d) Using gamma functions, evaluate

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}}$$ 3

(e) Prove that

$$\beta(m, n) = \frac{\Gamma(n)\,\Gamma(m)}{\Gamma(m+n)}$$ 4

4. (a) If $f(z) = w = u(x, y) + iv(x, y)$ is analytic in a domain, then in that domain

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y};\ \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

Prove this statement. 4

(b) Using Cauchy's integral formula, calculate $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is the circle $|z| = 2$ described in the positive sense. 2

(c) Evaluate : 3

$$\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$$

Or

Obtain the Laurent series expansion of

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

in the region $1 < |z| < 2$. 3

5. (a) Find a series of sines and cosines which represents $x + x^2$ in the interval $-\pi < x < \pi$. Deduce that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$ 5

(b) Answer any *two* from the following : 3×2=6

(i) Find the Fourier series of the function $f(x) = x + \pi$, if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

(ii) Find the series of sines of multiples of x which represents x in the interval $\pi \geq x \geq 0$.

(iii) Write down the Fourier series in complex form. Establish the relationship between the coefficients of the complex form with a_0, a_n and b_n.

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