Total No. of Printed Pages—6

5 SEM TDC PHY M 1

2014

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×6=6

(a) A second-order differential equation is written as

$$m\frac{d^2y}{dt^2} = -ky - l\frac{dy}{dt} \quad (l > 0)$$

The solution will be underdamped or oscillatory, if

(i) $b^2 > \omega^2$

(ii) $b^2 = \omega^2$

(iii) $b^2 < \omega^2$

(b) The value of

$$\int_{-1}^{+1} xP_n \frac{dP_n}{dx} dx$$

is

(i) 0

(ii) $\frac{2}{2n+1}$

(iii) $\frac{2n}{2n+1}$

(c) The real part of an analytic function $f(z)$ is $u(x, y) = x^2 - y^2$. $f(z)$ is equal to

(i) z

(ii) z^2

(iii) $|z|^2$

(d) The sum of residues of the function

$$f(z) = \frac{e^z}{z^2 + a^2}$$

at all its poles is

(i) $\frac{\sin a}{a}$

(ii) $-\frac{\sin a}{a}$

(iii) $\frac{\cos a}{a}$

(e) Which of the following identities is not correct?

(i) $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

(ii) $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

(iii) $\frac{\pi}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} -$

(f) The value of the Fourier coefficient a_0 in the Fourier series of t^2 in the interval $-\pi < t < \pi$ is

(i) 0

(ii) $\frac{\pi^2}{3}$

(iii) $\frac{\pi^2}{8}$

2. (a) The rate at which a radioactive substance decays is proportional to the remaining number of atoms. If there are N_0 atoms at $t = 0$, find the number at time t. 2

(b) Solve the initial value problem : 2

$$y'' - y = 0, y(0) = 4, y'(0) = -2$$

(c) Given $w = f(z) = z^2 + 3z$. Find u and v, and calculate the value of f at $z = 1 + 3i$.

½+½+1=2

(d) Find the Maclaurin series of

$$f(z) = \frac{1}{1+z^2}$$ 2

(e) What are the conditions that a function must satisfy so that the Fourier series of the function converges uniformly? 2

(f) What are Fourier sine and cosine series? 2

3. (a) Find a general solution of

$$y'' - 3y' + 2y = e^x$$ 3

Or

Prove that

$$(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$$

(b) Find the power series solution of the differential equation : 5

$$y'' + xy' + (x^2 + 2)y = 0$$

Or

Solve the initial value problem :

$$y'' + 2y' + 101y = 10 \cdot 4e^x,\ y(0) = 1 \cdot 1,\ y'(0) = -0 \cdot 9$$

(c) Set up the equation of motion for forced vibrations of a damped oscillator. Solve the equation of motion. Discuss the case of resonance and derive the form of the solution in the case of resonance. 1+2+2+2=7

4. (a) Prove that

$$P_n(x) = \frac{1}{2^n\, n!}\left(\frac{d}{dx}\right)^n (x^2 - 1)^n$$ 5

(b) Prove that

$$\beta(m, n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1} (\cos\theta)^{2n-1}\, d\theta$$ 3

Or

Find :

$$I = \int_0^{\infty} \frac{x^3\, dx}{(1+x)^5}$$

(c) A sinusoidal voltage $E \sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function

$$u(t) = \begin{cases} 0 & \text{if } -L < t < 0 \\ E \sin \omega t & \text{if } 0 < t < L \end{cases} \quad p = 2L = \frac{2\pi}{\omega}, \quad L = \frac{\pi}{\omega}$$

5

5. (a) Given

$$f(x) = \begin{cases} 0 \;, & 0 < x < l \\ 1 \;, & l < x < 2l \end{cases}$$

Expand $f(x)$ in an exponential Fourier series of period $2L$. 5

(b) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace's equation and determine the corresponding regular function $u + iv$. 5

Or

Using Cauchy's integral formula, evaluate the integral $\oint \frac{z^2}{(z^2 - 1)} dz$ around the unit circle with centre at *(i)* $z = 1$, *(ii)* $z = -1$ and *(iii)* $z = \frac{1}{2}$. 2+2+1=5

(c) Find the residue of the following functions at the mentioned poles : 2+2=4

(i) $\frac{\cos z}{z}$ at $z = 0$

(ii) $\cot z$ at $z = 0$

★★★