Total No. of Printed Pages—8

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(November)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

(*Marks* : 27)

1. (a) What will be the equation of the circle $(x-h)^2+(y-k)^2=r^2$, when the origin is transferred to the point (h, k)? 1

(b) Prove that if $ax^2 + 2hxy + by^2 = 1$ and $a'x'^2 + 2h'xy + b'y^2 = 1$ represent the same conic and the axes are rectangular, then show that

$$(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$$ 2

(c) The axes are rotated through an angle 60° without changing the origin. If the coordinates of a point are $\left(\frac{1}{2}, \frac{5\sqrt{3}}{2}\right)$ in old system, what would be its coordinates in new system? 2

2. (a) State the name of the geometrical figure represented by the equation $xy = 0$. 1

(b) Find the equations of the straight lines which pass through the origin and whose distance from (h, k) are equal to d. 2

(c) Prove that the lines represented by

$$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$$

have the same pair of bisectors for all values of λ. Interpret the case for $\lambda = -(a+b)$. 2+2=4

Or

If the straight lines represented by the equation

$$x^2(\tan^2\phi+\cos^2\phi)-2xy\tan\phi+y^2\sin^2\phi=0$$

make angles α and β with the x-axis, show that $\tan\alpha-\tan\beta=2$. 4

(d) Find the condition that one of the lines given by $ax^2+2hxy+by^2=0$ may be perpendicular to one of the lines given by $a'x^2+2h'xy+b'y^2=0$. 5

Or

Prove that the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents a pair of parallel straight lines if $\frac{a}{h}=\frac{h}{b}=\frac{g}{f}$.

3. (a) State True or False : 1

When the focus lies on the directrix, the conic section is a pair of lines.

(b) Find the equation of the polar of the point (2, 3) with respect to the conic

$$x^2+3xy+4y^2-5x+3=0$$ 2

(c) Find the condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ may be conjugate diameter of the conic

$$ax^2 + 2hxy + by^2 = 1$$ 3

Or

Find the equation of the diameter of the conic $15x^2 - 20xy + 16y^2 = 1$ conjugate to the diameter $y + 2x = 0$.

(d) Find the equation of the chord of contact of tangents from a given point (x_1, y_1) to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$ 4

Or

Reduce the equation

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

to the standard form.

SECTION—II

(3-Dimension)

(*Marks* : 18)

4\. (a) If O is origin and $P(a, b, c)$, then write the direction cosines of the line OP. 1

(b) A plane cuts the axes at A, B, C and the centroid of the triangle is (a, b, c). Find the equation of the plane. 3

Or

Find the equation of the plane passing through the point $(3, -2, 6)$ and through the x-axis.

(c) Find the equations of the line passing through the point $(2, 1, 0)$ and parallel to the line joining the points $(1, 5, 2)$ and $(3, 0, -1)$. 2

(d) Put the equations

$$4x - 4y - z + 11 = 0 = x + 2y - z - 1$$

of a line in the symmetrical form. 4

Or

Find the distance of the point $(-3, 1, 1)$ from the plane $2x + y - 4z + 6 = 0$ measured parallel to the line

$$\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$$

5. (a) Fill in the blank : 1

The shortest distance between the two lines is their common ____.

(b) Find the shortest distance between the line

$$ax + by + cz + d = 0 = a'x + b'y + c'z + d'$$

and the z-axis. 3

(c) Show that the shortest distance between the lines $x - y + z = 0 = 2x - 3y + 4z$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$ is $\frac{13}{\sqrt{66}}$. 4

Or

Find the length and the equations of the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \text{ and } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

GROUP—B

(Algebra—I)

(*Marks* : 35)

6. (a) State True or False : 1

Addition of natural number in binary composition is not associative.

(b) Define a quaternion group. 1

(c) Show that a finite semigroup in which cross-cancellation holds is an Abelian group. 2

(d) Answer any *two* : $3\times2=6$

(i) Show that a subgroup of a cyclic group is cyclic.

(ii) Let H, K be subgroup of G. Show that HK is a subgroup of G if and only if $HK = KH$.

(iii) Show that for elements a, b in a group G, the equations $ax = b$ and $ya = b$ have unique solutions for x and y in G.

7. Answer any *two* : $5\times2=10$

(a) If a group has finite number of subgroups, then show that it is a finite group.

(b) If G be a group and $a, b \in G$, such that (i) $ab = ba$ and (ii) $(O(a), O(b)) = 1$, then show that $O(ab) = O(a)O(b)$.

(c) Prove that a nonempty subset H of a group G is a subgroup of G, iff—

(i) $a, b \in H \Rightarrow ab \in H$;

(ii) $a \in H, a^{-1} \in H$.

8. (a) Define a simple group. 1

(b) Prove that every quotient group of a cyclic group is cyclic. 4

Or

If G is a group such that $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is centre of G, then show that G is Abelian. 5

(c) Answer any *two* : 5×2=10

(i) Show that a subgroup H of a group G is normal in G if and only if

$$g^{-1}hg \in H \quad \forall h \in H, g \in G$$

(ii) If H and K be two subgroups of a group G, where H is normal in G, then prove that

$$\frac{HK}{H} \cong \frac{K}{H \cap K}$$

(iii) Find the regular permutation groups isomorphic to the multiplicative group $G = \{1, -1, i, -i\}$.

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