

Total No. of Printed Pages—8

3 SEM TDC MTH M 1

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(November)

MATHEMATICS

(Major)

Course : 301

[Analysis—I (Real Analysis)]

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Differential Calculus)

(Marks : 35)

1. (a) If $y = \log(a + x)$, find y_n . 1

(b) If $y = e^{3x} \sin 4x$, find y_n . 2

(c) Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$$

3

Or

Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

(d) If $y = (\sin^{-1} x)^2$, then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

4

Or

Find the radius of curvature at any point (x, y) for the curve $y = \log \sin x$.

2. (a) Write the geometrical interpretation of the Lagrange's mean value theorem. 2

(b) State and prove Rolle's theorem. 4

(c) Expand e^x in a finite series in powers of x with Lagrange form of remainder. 4

Or

If $f'(x) = 0$ for all values of x in an interval, then show that $f(x)$ is constant in that interval.

3. (a) Find $\frac{\partial f}{\partial x}$, where $f = e^{x^2 + xy}$.

1

(b) If

$$u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$$

then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

4

Or

If

$$u = \sin^{-1} \frac{x^2 + y^2}{x + y}$$

then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.

4. (a) Define Jacobian of a function of two variables.

1

(b) Write the necessary condition for a function $f(x, y)$ to have an extreme value at (a, b) .

1

(c) If

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

then show that the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ exist at $(0, 0)$.

3

- (d) Find the maximum and minimum value of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20 \quad 5$$

Or

If $v = v(x, y)$ and $x = r \cos \theta$, $y = r \sin \theta$,
then find $\frac{\partial^2 v}{\partial x^2}$.

GROUP—B

(Integral Calculus)

(Marks : 20)

5. (a) Write the condition, when $\int_0^{2a} f(x) dx = 0$. 1

- (b) Show that

$$\int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx \quad 2$$

- (c) Evaluate any one : 3

(i) $\int_0^{\pi/2} \log \tan x dx$

(ii) $\int_0^1 \frac{x^6}{\sqrt{1-x^2}} dx$

- (d) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x dx \quad 4$$

Or

Evaluate

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$$

6. (a) Write the area of the surface of the solid obtained on revolving about x -axis, the arc of the curve $y = f(x)$ intercepted between the points whose abscissas are a and b . 2

- (b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum. 4

- (c) Find the volume of the solid obtained by revolving one arc of the cycloid

$$x = a(\theta + \sin \theta), \quad y = a(1 + \cos \theta) \quad \text{4}$$

Or

Find the surface generated by the revolution of an arc of the catenary

$$y = c \cosh \frac{x}{c}$$

about x -axis.

GROUP—C

(**Riemann Integral**)

(Marks : 25)

7. (a) Write the condition when the function f is Riemann integrable over $[a, b]$. 1

(b) Choose the correct answer for the following : 1

If $\int_a^b f(x) dx$ exists, then

(i) f is bounded

(ii) f is unbounded

(iii) interval $[a, b]$ is finite

(iv) Both (i) and (iii)

(c) Show that x^2 is integrable on any interval $[0, a]$. 3

(d) Prove that every continuous function is integrable. 3

Or

If P_1 is a refinement of a partition P , then for a bounded function f , show that $U(P_1, f) \leq U(P, f)$.

8. (a) Prove that if a function f is continuous on $[a, b]$, then there exists a number c in $[a, b]$, such that

$$\int_a^b f dx = f(c)(b-a) \quad 4$$

- (b) If f is continuous and positive on $[a, b]$, then show that $\int_a^b f dx$ is also positive. 3

Or

Explain the Riemann integrability of

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

9. (a) $\int_0^\infty \frac{dx}{1+x^2}$ is an improper integral. Justify why it is an improper integral. 1
- (b) Write the statement of Abel's test. 1
- (c) Show that

$$\int_0^\infty \sin x^2 dx$$

is convergent.

4

Or

Test the convergence of the integral

$$\int_0^{\infty} \frac{\cos x}{1+x^2} dx$$

- 10.** (a) Show that $B(m, n) = B(n, m)$. 2
- (b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 2
