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2 PGDE MTH 2

2012

(Nov.-Dec.)

MATHEMATICS

Paper : 202

(Measure Theory and Computer Programming)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

(Measure Theory)

1. Answer any *two* of the following questions :
4×2=8

(a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that
$$m(\cup A_n) \leq \sum m(A_n).$$

(b) Prove that if E_1 and E_2 are two measurable sets, then $E_1 \cup E_2$ is also measurable.

- (c) Let $\langle E_n \rangle$ be an infinite decreasing sequence (ie $E_{n+1} \subset E_n$ for each n) of measurable sets and $m(E_1)$ is finite. Then prove that

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \rightarrow \infty} m(E_n)$$

2. Answer (a) and (b) or (c) :

(a) Show that the collection of Borel sets is the smallest δ -algebra which contains all of the open sets. 4

(b) Prove that every Borel set is measurable. 4

(c) Prove that there exists a non-measurable set in the interval $[0, 1]$. 8

3. Answer any *two* of the following questions :

(a) Let f and g are measurable functions and c is a constant. Then prove that the functions $f+c$ and $f+g$ are measurable. 2+2=4

(b) If f is a measurable function and $f = g$ a.e., then prove that g is measurable. 4

(c) Prove that sum and product of two simple functions is simple. 2+2=4

4. Answer either (a) or (b) :

(a) State and prove Lusin's theorem. 8

(b) If the sequence $\{f_n\}$ converges in measure to f , then prove that

(i) it converges in measure to every function g which is equivalent to the function f . 4

(ii) the limit function is unique a.e. 4

5. Answer either (a) or (b) and (c) :

(a) A bounded function is Riemann integrable if and only if it is continuous almost everywhere. 8

(b) If ϕ and ψ are simple functions which vanish outside a set of finite measure then prove that $\int a\phi + b\psi = a \int \phi + b \int \psi$, for all reals a and b . 4

(c) State and prove the bounded convergence theorem. 4

(Computer Programming)

6. Answer any *two* of the following :

- (a) Write a note on algorithm. 3
- (b) Draw a flow chart to pick up the largest of 3 numbers. 3
- (c) Write an algorithm to determine the greatest of three given numbers. 3

7. Write the functions of the printf () and the scanf () function giving suitable example. 5

Or

Write a program to generate the following output. 5

```
1 2 3 4 5
  2 3 4 5
    3 4 5
      4 5
        5
```

8. Using logical AND and logical OR write a program to print the numbers between 100 and 200 which are divisible by 4 but not divisible by 8. 5

Or

Write a program to enter a line of text through the keyboard and display the same, then convert it to upper case alphabet and display the output.

9. Write a program to implement Newton-Raphson method or trapezoidal rule. 6

10. The gross pay of an employee of an organisation is computed as below : 6

(i) If the basic pay is less than Rs. 5000, then House Rent Allowance is Rs. 2500, Dearness Allowance is 65% of basic pay and grade pay is Rs. 2000.

(ii) If the basic pay is Rs. 5000/- or more, then House Rent Allowance is Rs. 4800, Dearness Allowance is 50% of the basic pay and grade pay is Rs. 4000.

(iii) All employees get Rs. 350 as medical allowance.

Gross pay = Basic pay + House Rent Allowance + Dearness allowance + Grade pay + Medical allowance.

Write a program for it.

Or

Write a program to determine the selling price of an item to make a profit of 40% or Rs. 200 whichever is less.

11. Write a program to print the sum of a given list of numbers. 6

Or

Write a program to read two matrices and to compute the sum of the matrices.

12. Write a function to compute $x^3 - 7 \log x + 5e^x$ and use it in main () to print the values of the function for $x = 1, 1.1, 1.2, \dots, 2.0$. 6

Or

Write a program to initialize an array of numbers $x_i = 1.5, -1, 3.5, 2.75, -1.75, 3.75, 3.05, 4.95, -1.05, 2.55$ and to compute and print the sum of the numbers.