Total number of printed pages–8

2 PGDE MTH 2

2013

(December)

MATHEMATICS

Paper : 202

Measure Theory and C.P.

(New Course)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Write the answers to the ***separate Halves*** *in* ***separate books.***

FIRST HALF

(*Measure Theory*)

Marks : 40

1. Answer ***any two*** of the following questions : 4×2=8

   (a) Prove that M the class of measurable sets is closed under constable union.

(b) Show that the empty set is measurable.

(c) Is the set A given by $A=\left\{x\in \mathbb{R} \,\middle|\, x^2=1\right\}$ measurable ? Justify your answer.

2. Answer ***any two*** of the following questions : $4\times2=8$

(a) Show that the Control Set C is of measure O.

(b) Define a Borel set. Give two examples of Borel Sets with justification.

(c) If E is a measurable set then show that given $\in>0$, there is an open set $O\supset E$ such that $m^*(O-E)<\in$, symbols have their usual meanings.

3. Answer ***any two*** questions from the following : $4\times2=8$

(a) Show that the function ψ defines on $\mathbb{R}$ by

$$\psi(x)=\begin{cases} x+5, & \text{if } x<-1 \\ 2, & \text{if } -1\le x<0 \\ x^2, & \text{if } x\ge 0 \end{cases}$$

is measurable.

(b) Show that measurability of the set $\{x \mid f(x) = \alpha\} \forall \alpha \in \mathbb{R}$ does not imply measurability of f.

(c) Prove that a continuous function is measurable.

4. Answer *either (a)* **or** *(b)* 8

(a) State and prove Egoroff's theorem.

(b) State and prove Fréchet Theorem.

5. Answer ***any two*** questions from the following : $4 \times 2 = 8$

(a) Define a simple function. Define Lebesgue Integral of a simple function. Find the Lebesgue integral of the following simple function $f : [0, 4] \to \mathbb{R}$

$$\begin{aligned} f(x) &= 2 + x \quad \text{if} \quad x \in [0, 1) \\ &= 2x \quad \text{if} \quad x \in [1, 2) \\ &= 3x \quad \text{if} \quad x \in [2, 3) \\ &= 4x \quad \text{if} \quad x \in [3, 4] \end{aligned}$$

(b) Give an example of a function which is Lebesgue Integrable but not Riemann integrable. Justify your answer.

(c) State and prove the Bounded Convergence Theorem of Lebasgue Integration.

SECOND HALF

(*Computer Programming in C*)

Marks : 40

1. Answer ***any one*** question :

(a) (i) What is flow chart ?

(ii) Describe different symbols used in flow chart.

(iii) Draw the flow chart for obtaining the g.c.d. of two numbers. 1+2+3=6

(b) (i) Draw a flow chart to pick up the largest of 3 numbers. 3

(ii) Discuss the various data types in C giving examples is each. 3

2. Answer ***any one*** question :

(a) *(i)* What is a variable ?

(ii) What is meant by the value of a variable ?

(iii) Find Syntax errors in the following program : 1+1+3=5

```
# define PI 3.14159
int main ( )
        {
int R, C ; / * R– radius of circle */
float perimeter ;
float area ;
C = PI ;
R = 5
Perimeter = 2.0 * C * R ;
Area = C * R * R ;
printf ("%F, %d", & perimeter, & area);}
```

What would be the output of the program after correction.

(b) The variables count, price and city have the following values

count → 1275

price → – 235.74

city → Dibrugarh

Write the outputs that the following statement produce. 5×1=5

(i) printf (" %d %f", count, price) ;

(ii) printf (" % 2d \n % f ", count, price);

(iii) printf (" % d % f ", price, count) ;

(iv) printf (" % 10dxxxx % 5.2 f ", count, price);

(v) printf (" % S " , city) ;

(vi) printf (" % – 10d% – 15 S", count, city);

3. Answer ***any one*** question :

(a) Write a C program to compute and print the roots of the quadratic equation $ax^2 + bx + c = 0$. 5

(b) Using logical AND and logical OR, write a program to print the numbers between 300 and 600 which are divisible by 4 but not divisible by 8. 5

4. Write a 'C' program to compute a real root of the equation.

$$x - 0 \cdot 2 \sin x - 0 \cdot 5 = 0,$$

using Newton-Raphson method. The computation should stop either number of iteration greater than 10 or the roots should be corrected to six significant figures. 6

5. Answer ***any one*** question :

(a) The series 1, 1, 2, 3, 5, 8, 13, 21, ... is called Fibonacci series they are described by the recurrence relation given by,

$$T_0 = 1$$

$$T_1 = 1$$

$$T_n = T_{n-1} + T_{n-2} \text{ for } n \geq 2$$

Write a program to generate first 100 Fibonacci numbers. 6

(b) Write a program in 'C' given the cost price to determine the selling price of an item to make a profit of 20% or Rs. 250 whichever is less. 6

6. Answer ***any one*** question :

(a) Define a function prototype.
Why is a function prototype necessary ?
Write a function to evaluate the factorial of a natural number. 1+1+4=6

(b) Explain with example about a function with argument but no return. 6

7. Answer ***any one*** question :

(a) The different labourers are paid different amount of money for their engagement in work of different type in a project. Write a program to calculate total labour cost in the project. 6

(b) A $n \times m$ matrix is multiplied by a number K to get a new matrix. Write a 'C' program to read original matrix and print out the resulted matrix. 6