Total number of printed pages–5

2 PGDE MTH 1

2013

(December)

MATHEMATICS

Paper : 201

Topology & F. Analysis

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

(***Topology***)

Marks : 40

1. Answer ***any three*** questions : 3×4=12

   (a) Define *two* topologies on
   $$X = \{a, b, c\}$$

   (b) What is interior of a set ? Show that interior of a set is an open set.

   (c) Define limit point of a set. Show that if a set contains all its limit points then the set is closed.

(d) Show that a map $f : X \rightarrow Y$ is continuous iff the inverse image of any basic open set of Y is open in X.

2. Answer ***any three*** questions : $3 \times 4 = 12$

(a) Define a T_0-space. Show that a T_1-space is T_0.

(b) Show that a T_2-space is T_1.

(c) Define first and second countability spaces. Show that a second countable space is first countable.

(d) What is a Lindelof's space? Give an example.

3. Answer ***any two*** questions : $2 \times 4 = 8$

(a) Define compact space. Show that closed subset of a compact space is compact.

(b) Prove that the continuous image of a compact space is compact.

(c) Define finite intersection property (FIP). Show that in a compact space every finite of closed sets with FIP has non-empty intersection.

(d) Is $\mathbb{R}$ with usual topology compact ? Justify.

4. Answer ***any two*** questions : $2\times4=8$

(a) Define separated sets.
Show that disjoint sets need not be separated.

(b) Let X be a connected space and $f : X \to \mathbb{R}$ be a continuous function. Show that f has intermediate value property.

(c) Define component. Show that components are closed sets.

(*Functional Analysis*)

Marks : 40

5. Answer ***any one*** question : 8

(a) Let X and Y be normed linear spaces and $T : X \to Y$ be linear. When T is said to be bounded ? Show that T is continuous iff T is bounded.

(b) Let X and Y be normed linear spaces and $B(X, Y)$ denote the set of all bounded linear operators from X into Y. Show that $B(X, Y)$ is a normed linear space.

6. Answer ***any one*** question : 8

(a) Define normed linear space and Banach space. Give example of a normed space which is not a Banach space.

(b) Let M be a closed subspace of a Banach space X. Define a suitable norm on X/M. So that it is also a Banach space.

7. Answer ***any one*** question : 8

(a) What are linear functionals ? Give example. Explain the concept of dual space with suitable example.

(b) State and prove the Hahn-Banach theorem.

8. Answer ***any one*** question : 8

(a) Define inner-product space and Hilbert space. Let X be a Banach space whose norm satisfies the parallelogram law. Show that X is a Hilbert space.

(b) What is an orthonormal basis? Show that every finite dimensional inner product space has an orthonormal basis.

9. Answer ***any one*** question : 8

(a) Define the following :

Hilbert adjoint operator, self-adjoint operator, normal operator and unitary operator.

(b) Let H be a Hilbert space and $T, T_1, T_2 \in B(H)$. Prove that

$$(T_1 + T_2)^* = T_1^* + T_2^*$$

$$(\alpha T)^* = 2T^*, \quad \alpha \text{ is a scalar}$$

$$\|T\| = \|T^*\|$$