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(May)

MATHEMATICS

(Major)

Course : 201

(Matrices, Ordinary Differential Equations, Numerical Analysis)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Matrices)

(*Marks* : 20)

1. (a) Write True or False : 1

"If all $(r+1)$ rowed minors of a matrix vanish, the rank of the matrix is $\leq r$."

(b) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$$ 2

(c) Reduce the matrix

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

to echelon form and find its rank. 5

2. Answer any *two* of the following : 6×2=12

(a) Find the characteristic equation of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

and verify that it is satisfied by A and hence find its rank.

(b) Define eigenvalue. What is the eigenvalue of $P^{-1}AP$, if eigenvalue of matrix A is λ? Investigate for what values of λ and μ, the simultaneous equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions.

(c) What do you mean by homogeneous and non-homogeneous linear equation? Show that the following system of equations

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

is consistent.

GROUP—B

(Ordinary Differential Equations)

(*Marks* : 30)

3. (a) Find the Wronskian of the functions x^2 and e^x. 1

(b) Solve : 2

$$\frac{dy}{dx} + \frac{y - x^2}{x + y^2} = 0$$

(c) Solve : 3

$$x = y - p^2, \; p = \frac{dy}{dx}$$

(d) Answer any *one* of the following : 4

(i) If $y_1(n)$ and $y_2(n)$ are any two solutions of

$$a_0(n)y''(n) + a_1(n)y'(n) + a_2(n)y(n) = 0$$

then prove that the linear combination $e_1y_1(n) + e_2y_2(n)$, where e_1 and e_2 are constants, is also a solution of the given equation.

(ii) Show that linear independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ and $e^x \cos x$. Find the solution $y(n)$ with the property $y(0) = 2,\ y'(0) = -3$.

4. (a) Write True or False : 1

"For the equation $(D^2 + RD + S)y = 0$, $y = x$ is a particular integral, if $R + xS = 0$."

(b) Answer any *two* of the following : 2×2=4

(i) Solve :

$$\frac{d^2y}{dx^2} + (a+b)\frac{dy}{dx} + aby = 0$$

(ii) Find the particular integral of the differential equation

$$x\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y = 0$$

(iii) Solve :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$$

(c) If $y = e^x$ is a part of the CF, solve

$$x^2\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$ 5

Or

Solve : 5

$$(x^2D^2 + 7xD + 13)y = \log x, \ D \equiv \frac{d}{dx}$$

5. Answer any *two* of the following : 5×2=10

(a) Solve (by removing first-order derivative) :

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

(b) Solve (by changing the independent variable) :

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$$

(c) Solve (by the method of variation of parameters) :

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

GROUP—C

(Numerical Analysis)

(*Marks* : 30)

6\. (a) What is the length of the subinterval which contains x_n after n bisections? 1

(b) Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method. 5

Or

Solve by Gauss-Seidel method

$$10x - 5y - 2z = 3;\ 4x - 10y + 3z = -3;\ x + 6y + 10z = -3$$ 5

(c) Describe the Newton-Raphson method for obtaining the real roots of the equation $f(x) = 0$. 5

Or

Solve by Gauss elimination method

$$x + 2y + z = 3;\ 2x + 3y + 3z = 10;\ 3x - y + 2z = 13$$ 5

(d) Find an iterative formula to find $\sqrt{N}$, where N is a positive number and hence find $\sqrt{5}$. 4

7. (a) What is the degree of the polynomial in the trapezoidal rule? 1

(b) Find the relation between D and Δ, where $D =$ differential operator and $\Delta =$ forward difference operator. 2

(c) Evaluate : 2

$$\Delta\left(\frac{x}{\sin 2x}\right)$$

(d) Answer any *two* of the following : 5×2=10

(i) Deduce the Lagrange interpolation formula for unequal intervals.

(ii) Derive Simpson's $\frac{3}{8}$th rule for numerical integration.

(iii) Find the values of y at $x = 21$ and $x = 28$ from the following data :

x :	20	23	26	29
y :	0·3420	0·3907	0·4384	0·4848

(iv) Evaluate

$$\int_0^6 \frac{dx}{1+x^2}$$

by trapezoidal rule.

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