2 SEM TDC CSc G 1

2014

(May)

COMPUTER SCIENCE

(General)

Course : 201

(Discrete Structure)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Select the correct answer : 1×8=8

(a) In a relation R, if the presence of (a, b) excludes the possibility of presence of (b, a), then R is said to be

(i) reflexive

(ii) transitive

(iii) symmetric

(iv) asymmetric

(b) Which of the following functions is the generating function of the sequence 1, a, a^2, a^3, ... ?

(i) $G(x) = \frac{1}{1-x}$, $|x| < 1$

(ii) $G(x) = \frac{1}{(1-x)^2}$, $|x| < 1$

(iii) $G(x) = \frac{x}{(1-x)^2}$, $|x| < 1$

(iv) $G(x) = \frac{1}{1-ax}$, $|ax| < 1$

(c) The recurrence relation $a_n = 2a_{n-1}$ is a linear homogeneous relation with constant coefficients of degree

(i) 0

(ii) 1

(iii) 2

(iv) n

(d) If for a given size the complexity is taken as the maximum complexity over all inputs of that size, then the complexity is called the

(i) best-case complexity

(ii) average-case complexity

(iii) worst-case complexity

(iv) expected complexity

(e) A function $f(x) = a^x (a > 0)$ satisfying the law $a' = a$ and $a^x \cdot a^y = a^{x+y}$ is called the —— function.

(i) exponential

(ii) logarithm

(iii) rational

(iv) irrational

(f) A vertex of a graph of degree 1 is called

(i) isolated

(ii) pendent

(iii) adjacent

(iv) isomorphic

(g) An algorithm that makes an optimal choice at each of its steps without regard to previous choices is known as —— algorithm.

(i) Kruskal's

(ii) Prim's

(iii) DFS

(iv) greedy

(h) Let $C(x)$ denote 'x is clever', $S(x)$ denote 'x is successful'. The symbolic form of the sentence 'There are some students who are not clever' is

(i) $\exists x (S(x) \wedge C(x))$

(ii) $\exists x (\neg S(x) \wedge C(x))$

(iii) $\exists x (S(x) \wedge \neg C(x))$

(iv) $\exists x (\neg S(x) \wedge \neg C(x))$

2. Answer any *four* questions :

(a) Define proper subset. Prove that

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$ 1+3

(b) Define domain and range for a relation. The relation R on the set $\{1, 2, 3, 4, 5\}$ is defined by the rule $(x, y) \in R$ if 3 divides $x - y$. Find the elements of R.

2+2=4

(c) Discuss the various types of functions. 4

(d) Discuss the important notations used for describing growth of functions. 4

(e) What is a recurrence relation? Find the first four terms for the following recurrence relation :

$$a_k = 2a_{k-1} + k, \text{ for all integers } k \geq 2, \; a_1 = 1$$

1+3=4

(f) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 4

3. Answer any *eight* from the following :

(a) What is a power set? If $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{1, 2, 3, 4, 5\}$, find (i) $A \times B$, (ii) $C \times B$ and (iii) $B \times B$. Prove that

$$(C \times B) - (A \times B) = B \times B$$ 1+6=7

(b) Find the general solution to the following non-linear recurrence relation :

$$a_n - 5a_{n-1} + 6a_{n-2} = 1$$

Apply the generating function technique to solve the following recurrence relation :

$$a_{n+2} + 4a_{n+1} + 4a_n = 0 \; ; \; a_0 = 1, \; a_1 = 0$$

3½+3½=7

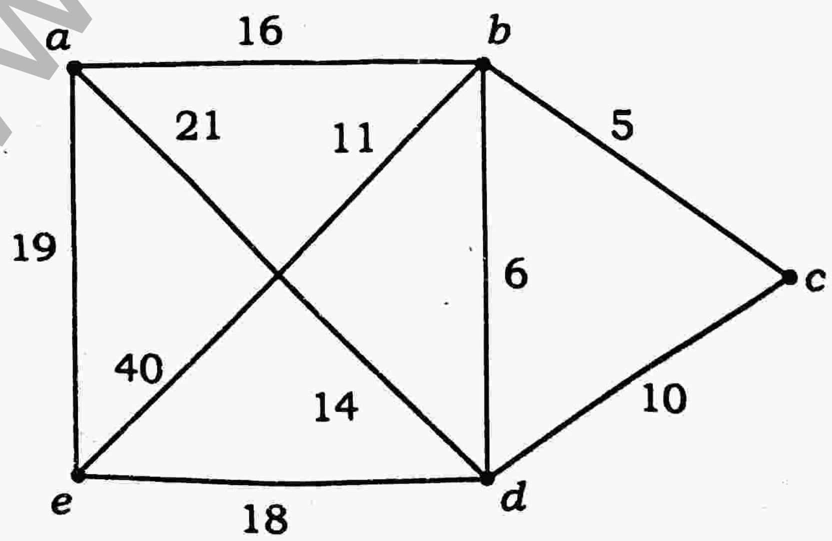
(c) Define primitive recursive function. Use the theorem on polynomial order to prove that $\frac{(x+1)(x+3)}{2}$ is $O(x^2)$. Write the value of *(i)* $\lfloor 8 \cdot 3 \rfloor$ and (ii) $\lceil 6 \cdot 0 \rceil$.

1+4+2=7

(d) Define Hamiltonian path and circuit. Explain the traveling salesman problem with an example. 2+5=7

(e) Define spanning tree. Use Krushal's algorithm to find a minimum spanning tree for the weighted graph given below :

2+5=7

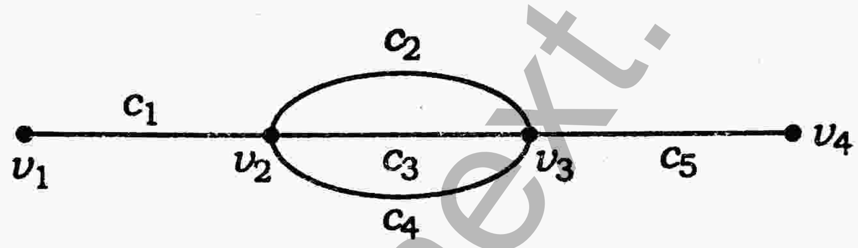


(f) Discuss BFS and DFS algorithms for spanning tree construction. Construct a binary tree whose in-order and pre-order traversal is given below : 4+3=7

In-order : 5, 1, 3, 11, 6, 8, 2, 4, 7

Pre-order : 6, 1, 5, 11, 3, 4, 8, 7, 2

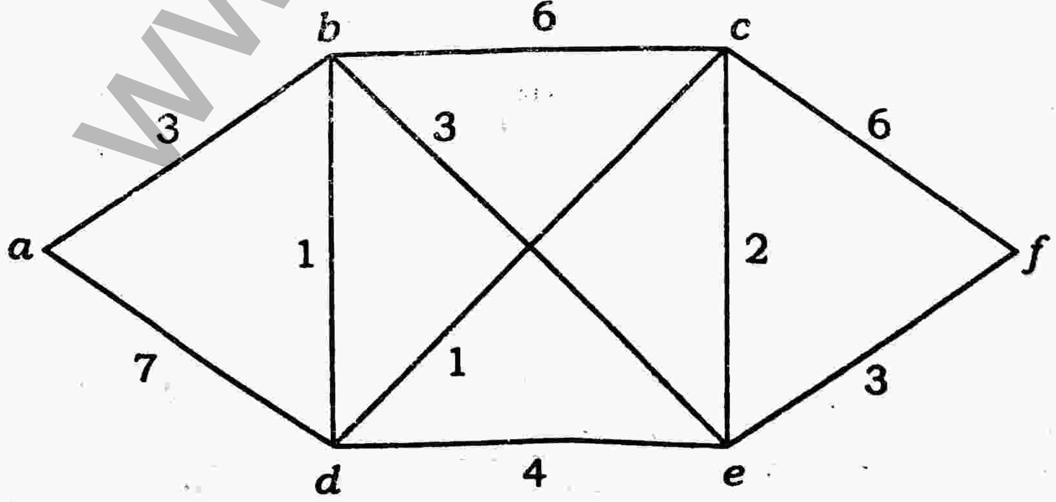
(g) (i) Consider the following graph :



(1) How many simple paths are there from v_1 to v_4?

(2) How many paths are there from v_1 to v_4? 3

(ii) Use Dijkstra's algorithm to find the shortest path between a and f in the weighted graph given below : 4



(h) What are universal and existential qualifiers? Define well-formed formula. Show that

$$\alpha = (P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \Rightarrow R))$$

is a tautology. 2+2+3=7

(i) How many integral solutions are there to the system of equations?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$
$$x_1 + x_2 = 15$$

where $x_k \geq 0$, $k = 1, 2, 3, 4, 5$. 7

(j) If the function $f : R \to R$ be defined by $f(x) = x^2 + 3$, find $f^{-1}(7)$ and $f^{-1}(19)$. 7

★★★