Total No. of Printed Pages—8

1 SEM TDC MTH M 1

2016

(November)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32 (Backlog) / 24 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the limit point of the sequence $\{1, 2, 1, 4, 1, 6, \cdots\}$. 1

(b) Write the condition under which every monotonic increasing sequence, which is not bounded above, diverges. 1

(c) Write when a sequence is called a bounded sequence. 1

(d) Choose the correct answer for the following : 1

A function whose domain is the set of natural numbers and ranges a set of real numbers is called

(i) limit of the function

(ii) derivative of the function

(iii) a real sequence

(iv) a complex sequence

(e) Is the sequence $\{1, 2, \frac{1}{2}, 3, \frac{1}{3}, \cdots\}$ bounded below and above? Write its limit points. 1+1=2

(f) Discuss the convergence of the sequence $\{S_n\}$, where

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}, \ \forall \, n \in N$$ 4

Or

Prove that a sequence cannot converge to more than one limit point.

2\. (a) Write the condition under which the infinite series Σu_n converges. 1

(b) Write an example of an alternating series. 1

(c) Write the statement of Cauchy's root test. 2

(d) Show that the series $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}}+\cdots$ is divergent. 3

(e) Show that the series $\sum \frac{1}{n}$ does not converge. 5

Or

Test the convergence of the series

$$\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\cdots$$

(f) Prove that a positive term series converges if and only if the sequence of its partial sums is bounded. 3

3. (a) Write the remainder when $x^5+x^4-3x^3+16$ is divided by $x+1$. 1

(b) Form the quadratic equation whose one root is $4-\sqrt{2}$. 2

(c) Find the nature of the roots of the equation $x^4+x^2+x-1=0$. 2

(d) Solve the equation $x^3-6x^2+11x-6=0$ when the roots are in AP. 3

Or

If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2\beta$ in terms of the coefficients of the equation.

(e) Find the equation whose roots are the reciprocals of the roots of the equation $x^4 - 4x^3 + 5x^2 + 11x - 12 = 0$. 2

(f) Solve $x^3 - 9x - 28 = 0$ by using Cardan's method. 5

Or

If α, β, γ are the roots of the equation $x^3 + px + q = 0$, form the equation whose roots are $\beta^2 + \gamma^2 - \alpha^2$, $\gamma^2 + \alpha^2 - \beta^2$, $\alpha^2 + \beta^2 - \gamma^2$.

GROUP—B

(Trigonometry)

4\. *(a)* Choose the correct answer for the following : 1

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ is true for

(i) only positive integral values of n

(ii) all real values of n

(iii) only positive real values of n

(iv) only integral values of n

(b) Write the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$. 1

(c) Find all the values of $(1+i)^{\frac{1}{5}}$. 3

Or

If n be a positive integer, prove that

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$$

(d) Expand $\cos^3 x$ in powers of x. 3

Or

Express $\cos 5x$ in powers of $\sin x$.

5\. (a) Show that

$$\log\sqrt{i} = \frac{1}{4}(8n+1)i$$ 2

(b) Show that

$$\log(1+i) = \log\left(2\cos\frac{\theta}{2}\right) + \frac{1}{2}i\theta$$ 3

6\. (a) Write the reason why Gregory's series cannot be applied to obtain the sum of the series $4 - \frac{1}{3}4^3 + \frac{1}{5}4^5 - \cdots$. 1

(b) Find the sum of the infinite series $1 - \frac{1}{3^2} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots$. 3

Or

Show that

$$\tanh x + \frac{1}{3}\tanh^3 x + \frac{1}{5}\tanh^5 x + \cdots = x$$

where $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

7. *(a)* Choose the correct answer for the following : 1

$\sinh^{-1} x$ is equal to

(i) $-i\sin^{-1}(ix)$

(ii) $i\sin^{-1}(ix)$

(iii) $-\sin^{-1}(ix)$

(iv) $-i\sin^{-1} x$

(b) If $\cos^{-1}(x+iy) = A + iB$, then show that

$$x = \cos A \cosh B$$ 2

(c) Separate $\sin^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts, where θ is a positive acute angle. 5

Or

Obtain the sum of the series

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \cdots$$

GROUP—C

(Vector Calculus)

8. (a) Let

$$\vec{A} = (x^2y^2 - x)\hat{i} + (e^{xy} - y\sin x)\hat{j} + x\cos y\hat{k}$$

Find $\dfrac{\partial \vec{A}}{\partial x}$. 1

(b) Show that

$$\vec{A} \cdot \frac{d\vec{A}}{dt} = A\frac{dA}{dt}$$

where $A = |\vec{A}|$. 2

(c) Define directional derivative of a scalar function. 1

(d) Let $\vec{v} = x^3y\hat{i} + 2y^2\hat{j} + x^2y^2z^2\hat{k}$. Find $\nabla \cdot \vec{v}$ at (1, 1, −1). 2

(e) If $\vec{v} = (-2x + y)\hat{i} + (x + ay)\hat{j} + (x + z)\hat{k}$ is a solenoidal vector, then determine the constant a. 2

(f) Prove that $\nabla f(r) = \dfrac{f'(r)\vec{r}}{r}$, $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and f is a differentiable function. 3

Or

Show that $\nabla r^n = nr^{n-2}\vec{r}$, $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(g) Prove that $\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 0$, where $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4

Or

Prove that $\nabla \times (\phi\vec{A}) = (\nabla\phi) \times \vec{A} + \phi(\nabla \times \vec{A})$, where $\vec{A}$ is a differentiable vector function and ϕ is a differentiable scalar function.

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